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INVOLUNTARY UNEMPLOYMENT EXPLAINED BY OVER-DETERMINATENESS

By Noboru Kamakura

1. Recent attempts to combine the notion of involuntary unemployment with the idea of overdetermined economic system

Among recent studies on the logical basis of involuntary unemployment, the attempt to explain it in terms of an over-determined economic system is of great interest. The methodological outline of the idea was given by Professor Haavelmo[1] and was applied to economic analysis by Professors Patinkin[2], Domar[3] and others[4].

The following passage quoted from Professor Patinkin's paper will clearly show what those authors have aimed at.

"Within the frame work of traditional Keynesian economics, there are at least two basic issues which have not yet been settled. The first centres about the frequently heard complaint that Keynesian models neglect the supply side of the market. The second is concerned with the very question which brought forth the General Theory: involuntary unemployment. Examination of the Keynesian theory shows that (even granted its argument) it explains primarily the level of employment; it is inadequate in providing either a criterion for the measurement of unemployment, or a justification for calling it *involuntary*[5]."

Thus pointing to the defect in the Kenyesian analysis, Professor Patinkin continues:—

1) T. Haavelmo, The Notion of Involuntary Economic Decisions, *Econometrica*, Jan. 1950.

2) D. Patinkin, Involuntary Unemployment and the Keynesian Supply Function, *The Economic Journal*, Sept. 1949.

3) E. D. Domar, Capital Accumulation and the End of Prosperity, *Proceedings of the International Statistical Conference*, 1947, vol. 5, 1949.

4) As an antecedent of this line of studies, Professor R. Frisch, who had analyzed the economic implication of over-determined system in other contexts, must be listed. R. Frisch, Overdeterminateness and Optimum Equilibrium, *Nordisk Tidskrift For Teknisk φkonomi*, 1948.

5) D. Patinkin, *ibid.*, p. 360.

"In the following sections, it will be shown that the two seemingly independent issues raised above are, in fact, vitally interrelated. In particular, it will be argued that the key to our difficulties lies in explicitly introducing supply functions into the standard Keynesian models: once this is done, the problem of defining and measuring involuntary unemployment is simultaneously solved[1]."

In short, their intention is to explain the involuntary unemployment by introducing the supply side into the Keynesian analysis; the factor to connect them being over-determinateness of the economic system.

2. Neglect of the supply side in the Keynesian theory

Mr. Keynes titled his main work *the General Theory of Employment and Money*, placing the emphasis on the adjective *general*. Thus he claimed that his doctrine was applicable to the general case including involuntary unemployment, which, he thought, had been excluded from the classical economic theory[2]. Under the classical economics, wage flexibility does not correct unemployment. The villain in this connection is the shortage of Effective Demand, because, in the Keynesian model, the level of employment is a function of Effective Demand, *i. e.* national income.

The Keynesian doctrine, in its simplest form, can be shown with the following three equations,

$$C=F(Y) \qquad (2.1)$$
$$I=G(Y,E) \qquad (2.2)$$
$$Y=C+I \qquad (2.3)$$

where (2.1) is the consumption function and (2.2) is the investment function, and they are behaviour functions. The equilibrium condition (2.3) represents the prin-

Table of Notations

C	consumption
I	investment
Y	national income
P	full employment capacity of production
K	capital equipment
N	level of employment
N_0	level of full employment
N^D	demand for labour
N^S	supply of labour
M	existing stock of money
L	demand for cash balance
O	supply of goods and services
D	demand for goods and services
E	exogenous factors
w	money wage rate
r	rate of interest
s	technical factors
t	time

1) D. Patinkin, *ibid.*, p. 361.

2) It is a question what should properly be meant by general theory. A criterion must be sought in assumptions on which the theory is built. In this sense one cannot take Mr. Keynes' analysis general.

ciple of Effective Demand, which is, according to Keynes, the substance of the General Theory of Employment[1]. In this simplified Keynesian model, the demand side takes more important part than the supply side. Once the levels of consumption and investment expenditures are given, then the level of national income is determined at the level of the sum of these expenditures. Since the level of employment is taken as a monotonly increasing function of national income in the Keynesian theory, the employment level simply depends on the demand side of the economic system. It is all natural, therefore, to hear so often the complaint that the Keynesian analysis neglects the supply side of market.

3. Professor Domar's explanation of an over-determined economic system

Professors Haavelmo, Patinkin and Domar attempt to introduce the supply side into the usual Keynesian model. For instance, Professor Domar adds the following three equations to (2.1)~(2.3) of last section[2].

$$Y=P \tag{3.1}$$

$$\frac{dK}{dt}=I \tag{3.2}$$

$$P=H(K,\ s) \tag{3.3}$$

Among these equations, (3.2) defines investment as the derivative of capital stock with respect to time. Equation (3.3) shows the level of output P, when existing stock of capital is fully utilized, under the given level of technique. This is a kind of social production function[3], in which full utilization of capital and full employment of labour are assumed. The former is explicitly stated in (3.3), whereas the latter is not. Equation (3.1), therefore, shows the condition that actual level of national income is at the level of full employment of both labour and capital.

These six equations have only five unknowns, Y, P, I, C and K. The system is clearly over-determined. The problem of our present concern is how to treat this over-determinateness. Among these six equations, (2.1)~(2.3) concern with the demand side of economy and (3.1)~(3.3) represent supply side of economy. The values of Y. C and I are determined by equations of demand side, and I being given, the values of K, P and Y are by those of the supply side. But there is no reason to assume equality between Y determined by the demand side and Y determined by the supply side. If the former Y is larger than the latter Y, there is the excess demand for aggregate products of the national economy. If, on the other hand, the first

1) J. M. Keynes, *ibid.*, p. 25.

2) There is a minor difference between this model and Domar's. But the difference is trivial.

3) The problem of aggregation is not investigated here.

is smaller than the second, there is the excess supply, and part of aggregate products can not be sold at the remunerative prices. In this case the amount of production must be decreased. In other words, when there is excess supply, equation (3. 1) must be given up, and the condition of full utilization of capital and full employment of labour can not be attained. This is an outline of Professor Domar's view on this problem.

4. A Reexamination of the Over-determined system

Professor Domar's case can be restated as follows :—

Variables contained both in demand side (2. 1)~(2. 3) and in supply side (3. 1)~(3. 3) in this model are I and Y. Over-determinateness, therefore, can be cured, if any one equation which contains I or Y is abandoned. I wish to examine cases of I and Y in turn.

i) The case of I. Equations containing I are (2. 2) and (3. 2), among which (3. 2) is nothing but a definitional relation. Hence, the only equation which can be abandoned is (2. 2). To abandon (2. 2) is to give up the behaviour function of investment. In this case, (2. 3) becomes identity, and investment is carried by entrepreneurs not to maximize their profit taking into account the level of national income *etc.* but to fulfil the gap between national income and consumption. It amounts to postulate Say's Law[1], which precludes over-determinateness and assures full utilization of capital and full employment of labour.

ii) The case of Y. Equations which contain Y are (2. 3) and (3. 1), among which (2. 3) can not be abandoned, because this relation $Y=C+I$, though it is an *ex-ante* equation, must be satisfied *ex-post*, too. The only equation to be omitted is (3. 1). It means, to give up the condition of full utilization of capital and full employment of labour.

These are the two alternatives to escape from over-determinateness. The way of dropping (2. 2) is to postulate Say's Law, which assures the full utilization of capital and the full employment of labour. But this is an unrealistic way, because then the behaviour function of investment disappears, and the amount of investment is fixed to fulfil the gap between national income and consumption at the full employment.

It is, therefore, the only proper way is to give up (3. 1). This explains the reason why there exists an involuntary unemployment.

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1) O. Lange, Say's Law: A Restatement and Criticism, *Studies in Mathematical Economics, In Memory of Henry Schultz*, 1942.

5. Involuntary unemployment and frictional unemployment

The underlying idea of this formal analysis is summarized by Professor Patinkin in the passage: "To say that the system is mathematically inconsistent is equivalent to saying that the desires of people, as reflected by this system, are incompatible: they cannot all be satisfied simultaneously[1]." No matter where the economy may be, some people must be off their behaviour curves and must be acting involuntarily.

In the passage cited above, the word "equivalent" must not be taken so strictly, because in another context Patinkin writes: "The existence of a consistent equilibrium position for the static system is a necessary, but not a sufficient condition for the elimination of involuntary action within the economy[2]."

Professor Patinkin's model contains nine equations[3],

$$D=\varphi(Y,\ r,\ p) \qquad (5.1)$$
$$Y=\psi(N^D) \qquad (5.2)$$
$$D=Y \qquad (5.3)$$
$$N^D=f(w/p) \qquad (5.4)$$
$$N^S=g(w,\ p) \qquad (5.5)$$
$$N^D=N^S \qquad (5.6)$$
$$L=L(Y,\ r,\ p) \qquad (5.7)$$
$$M=\text{constant} \qquad (5.8)$$
$$L=M \qquad (5.9)$$

These divides up the economy into three markets. The first triplet concerns with the market for final goods and services, the second triplet with the market for labour, and the last triplet with the market for money. As there are nine equations for nine unknowns, we can assume, for simplicity, that his model is consistent. It, therefore, has a solution. Let the solution value of employment be $N^D=N^S=\sigma$. In this model, Professor Patinkin defines σ as the level of full employment[4].

He assumes that a sudden disturbance in the economy causes a downward shift in the expenditure function (5.1). This sets up a whole chain of dynamic events. During this process there is no reason why any of the equilibrium conditions——(5.3), (5.6) and (5.9)——should be satisfied. We may assume, therefore, equilibrium in the finished goods and services market and the money market is quickly reestablished. In this case, the only pressure for continued movements of the variables comes from the failure in satisfying (5.6). "Correspondingly, as long as this equilibrium condition is unstatisfied, the level of employment is less than σ. Hence, by definition, there is involuntary unemployment within the system. The level of this unemployment will continue to fluctuate as the system tries to correct the

1) D. Patinkin, *ibid.*, p. 382.
2) D. Patinkin, *ibid.*, p. 382.
3) D. Patinkin, *ibid.*, p. 379.
4) This point is fully discussed in next section.

disequilibrium in the labour market[1]." This is the point of Professor Patinkin's arguments.

The question is why Professor Patinkin calls involuntary unemployment the employment caused by the maladjustment in the labor market. As is well known, frictional unemployment is defined as that which is caused by various maladjustments, for example, unemployment due to a temporary want of specialized workers, intermittent changes in demand, time-lags in adjustment or the fact that the change-over from one employment to another can not be effected without some delay[2]. In short, "unemployment between jobs" which always exists in a non-static society, can be properly called as frictional unemployment. Professor Patinkin, however, calls this kind of unemployment as involuntary. But, the present writer does not believe that there are good reasons for calling it *involuntary*.

6. Norm of reference in Professor Patinkin's definition of involuntary unemployment

Let us investigate Professor Patinkin's definition of involuntary unemployment. Professor Patinkin writes, "The extent of involuntary unemployment is then measured by the difference between the existing amount of employment, and the amount that would have existed under norm" and "In what follows our norm of reference is defined as a model in which perfect competition reigns and the economic unit is restricted only by the budget restraint and technological relationship (*e. g.*, the production function). Thus, by definition, our norm is a system of equations. Within this norm of reference the individual will be defined as fulfiling his desires – though he may be poor and unhappy[3]."

Thus he emphasizes that it is theoretically meaningless to speak of involuntary unemployment without introducing a comparison between two alternative models: the actually existing one and some designated norm[4]. The system of equations (5. 1)~(5. 9) clearly defines the level of full-employment by his definition. In this sense, he takes the system (5. 1)~(5. 9) as norm of reference. Then we can not agree with him when he takes a dynamic process of adjustment as an alternative model to this norm of reference. The latter is static solution of a system and the former is dynamic process of

1) D. Patinkin, *ibid.*, pp. 380—381.

2) *cf.* J. M. Keynes, *ibid.*, p. 6.

3) D. Patinkin, *ibid.*, p. 369.

4) Professor Haavelmo also proposes that the concept of involuntary economic decision must be related to the comparison of alternative economic systems, and not to the decisions within a given system. *cf.* T. Haavelmo, *ibid.*, p. 2.

the same system when a sudden disturbance has shifted any function in the system. It is not easy to understand the reason why these two are properly called as *two alternative systems.*

7. Another difficulty in Professor Patinkin's definition

There is another difficulty in Professor Patinkin's definition of the level of full employment. As is already pointed out $N^D=N^S=\sigma$ is defined as the level of full employment, because this is the solution of a model in which perfect competition reigns and the economic unit is restricted only by the budget restraint and technological relationships.

Now assume a down-ward shift in (5. 1), then the amount of employment clearly decreases. A new level of employment can be known by comparative static method. Let it be $N^D=N^S=\sigma-\Delta\sigma$. Unemployment $\Delta\sigma$ caused by this is called as involuntary unemployment in the traditional Keynesian terminology, because it is caused by the decrease in Effective Demand.

But by Professor Patinkin's definition, the new level $N^D=N^S=\sigma-\Delta\sigma$ is the amount of full employment, because it also is a solution of a model in which perfect competition reigns and the economic unit is restricted only by the budget restraint and technological relationships. Therefore, this $\Delta\sigma$ can not be taken as involuntary unemployment by Professor Patinkin's definition.

8. The case of over-determinateness reexamined

Let us reexamine the case of over-determinateness. The point here is that the idea of over-determinateness is not only unnecessary but also unuseful to clarify the notion of involuntary unemployment. One reason is that over-determitateness is not a necessary condition for involuntary unemployment as already pointed out by Professor Patinkin and others. Another reason is that it will do without such a bewildering concept as over-determinateness for grasping the fact of involuntary unemployment. The reason is the following.

Let us introduce one more variable O to Professor Domar's model. Then over-determinateness is eliminated, as there are six equations for six unknowns. Profcssor Domar's model, therefore, is modified as follows:—

The question arises about the relation between O and Y. Three cases can be classified.

[Case I] $Y=O$. In this case aggregate demand is equal to aggregate

supply and there is full employment of labour and full utilization of capital. But there is no reason to expect $Y=O$ always holds.

[Case II] $Y>O$. This corresponds to the case of true inflation. Effective demand is so abundant and the production capacity is so limitted that the price level must rise up. Let us define Y_o as the level of Y when $Y=O$ holds. Now if we introduce the price level p as monotonous increasing function of Y,

$$C=F(Y) \tag{8.1}$$
$$I=G(Y,\ E) \tag{8.2}$$
$$Y=C+I \tag{8.3}$$
$$O=P \tag{8.4}$$
$$\frac{dK}{dt}=I \tag{8.5}$$
$$P=H(K,\ s) \tag{8.6}$$

$$p=p(Y),$$

and expand it in neighbourhood of Y_o and retain only the linear part of this expansion, we get

$$p-p_o=k[Y-Y_o],$$

where k is some positive constant coefficient and p_o is the initial price level. As $Y_o=O$ by definition, this can be rewritten as

$$p-p_o=k[Y-O]. \tag{8.7}$$

This is a kind of inflationary gap model.

[Case III] $Y<O$. Involuntary unemployment exists only in this case. Now let the employment function as monotonous increasing function

$$N=N[Y].$$

Expand this in neighbourhood of Y_o and retain only the linear part of this expansion, and we get

$$N_o-N=h[Y_o-Y]$$

where N_o is the number of would-be wage-earners in the system, and h is some positive constant coefficient. As $Y_o=O$, it can be rewritten as

$$N_o-N=h[O-Y]. \tag{8.8}$$

This is a model of involuntary unemployment in deflationary situations[1]. This explanation does seem to be more clear than that which employs the notion of over-determinateness.

9. Summary and an alternative proposal

When Professor Patinkin proposes explicitly to introduce a supply function into the usual Keynesian model, we can agree with him. When he

1) The relationship between (8.7) and (8.8) corresponds asymmetricity to inflation and deflation in the Keynesian doctrine. *cf.* J. M. Keynes, *ibid.*, p. 291.

criticizes the Keynesian economics by saying that it is inadequate in providing either a criterion for the measurement of unemployment or a justification for calling it involuntary, we can again agree with him.

But when he says that the problem of defining and measuring involuntary unemployment can be solved by introducing the supply function into the usual Keynesian models, the present writer can not agree with him. The notion of over-determinateness is neither necessary nor useful to explain involuntary unemployment. Definition of full employment as a solution of employment in the system, in which prefect competition reigns and the economic unit is restricted only by the budget restraint and technological relationships, does not seem to be adequate.

The present writer proposes to employ an idea of "dependency of market" for clarifying the notion of involuntary unemployment. It is useful to classify markets into "independent market" and "dependent market."

There is no need to say that there is the mutual inter-dependency of various markets in the economic system. The demand for and the supply of wheat, for instance, are brought into equilibrium by so many equilibrating factors. But the most important equilibrating factor for wheat market is the price of wheat. This is, of course, the usual type of market and the present author wishes to call such a type of market as an independent one.

In the Keynesian model, the equilibrating factor for labour is neither money rate of wage nor real rate of wage. Employment is a function of national income. In other words, the Keynesian type of labour market is solly dependent on the market for final goods and services, in the sence that once equilibrium is established in the market for final goods and services, then amount of employment is fixed automatically corresponding to the level of national income. Such a type of market may be called as a "dependent market." As to dependency of labour market in the Keynesian model, we may qnote a passage from Professor Takata's article.[1)]

"From Mr. Keynes' point of view, the amount of labour employed is not determined by labourers themselves comparing utility of wage and disutility of labour. It is solly dependent on other factors. Therefore, there remain labourers, who estimate their marginal utility of wage higher than marginal disutility of labour, unemployed. Involuntary unemployment exists in such a case."

According to the classical view on the labour market, employmeut and the wage rate are simultaneously determined when market equilibrium is established after higgling and bargaining between suppliers and demanders

1) Y. Takata, *Shin-Rishiron-Kenkyu (Studies in The New Theories of Interest)* 1940, p. 126.

of labour. The Keynesian view on the labour market differs basically from this in that the amount of labour employed, in the Keynesian system, is not determined by higgling and bargaining of both sides of labour market, but by factors outside the labour market: *e. g.*, national income. The characteristic features of the Keynesian labour market is its dependency on other markets.

An alternative proposal is, in short, that the concept of involuntary economic decisions, an example of which is involuntary unemployment, must be related to dependency of one market on other market.